Consequences of Skill: The Case of Abacus Training in Taiwan

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A general orientation toward the study of skills and their development is outlined, in which analyses of representation, transfer, and context are used to explore the consequences of developing a specific skill. This general approach is then applied to the study of abacus training and its implications for school achievement and cognitive development. Previous studies of abacus skill are reviewed, and two new studies are reported. Previous research showed abacus training to result in qualitative changes in children's representations of mental calculation through development of a "mental abacus." In Study 1, mental abacus skill was found to develop primarily as a result of practice rather than of selection factors such as socioeconomic status, ability, and previous mathematical knowledge. Abacus skill did, however, have positive effects on future achievement. Study 2 clarified the mechanisms underlying the functional relationships discovered in Study 1. Abacus training was found to affect both calculation skills and conceptual knowledge of the numeration system. The consequences of abacus training are both varied and limited. Consideration of effects of abacus training on the mental representation of mathematical knowledge, transfer of abacus skill to other tasks, and the contexts in which abacus skill develops demonstrates the multiple ways in which specific skills can contribute to cognitive development.

Introduction

Cultures vary in the repertoire of tools and knowledge that they provide their members. Such tools as calculators (Young 1981) or systems of

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navigation (Hutchins 1983) present users with powerful cognitive routines as well as subjects for thought. This paper explores the nature and cognitive consequences of a particular culturally transmitted tool, abacus calculation as practiced in Taiwan.

The abacus, a wood-framed tool composed of columns of movable beads, is used for arithmetic calculation throughout Asia. In Taiwan, where the present research was conducted, children study the abacus as part of the fourth- and fifth-grade mathematics curriculum. In approximately one hour per week of instructional time, children are taught the basics of addition, substraction, multiplication, and division using an abacus.

Whereas most children acquire a moderate level of skill in the operation of the abacus, some children (approximately 15 percent in the school in which this research was conducted) elect to participate in after-school abacus-training programs. These children, after about one year of training, develop what has been called a "mental abacus" (Hatano, Miyake, and Binks 1977; Stigler 1984). Children who reach this stage of training report being able to do mental calculation by first forming a visual image of an abacus and then moving the beads on their mental abacus exactly as they would on an actual abacus. Using this method, they are capable of extremely rapid and accurate mental calculation. For example, in an earlier study (Stigler 1984) fifth-grade abacus experts were found to be able mentally to add five three-digit numbers in about three seconds.

Children who pursue this additional abacus training use their skill primarily for competition, both national and international. Abacus training at this level is not related to the general mathematics curriculum of the elementary school. Rather, it is simply calculation for calculation's sake. What counts are speed and accuracy. Application to problem-solving situations is not a part of abacus training.

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What does it mean for children to acquire a skill such as mental abacus calculation? How is the skill acquired and represented mentally? What factors determine who becomes expert and who merely competent? What effect does it have on performance in other cognitive domains? How does this kind of mathematical knowledge relate to other aspects of mathematics that children are exposed to in school? These are the questions addressed in this paper.

Consideration of the significance of mastering a tool such as the abacus immediately raises a much more general question, namely, the role that acquisition of expertise in specific domains plays in the development of general cognitive abilities. If skills are the products of general abilities (e.g., Simon 1976), then cognitive development might best be studied by defining, measuring, and then charting the development of these general abilities, rather than by consideration of routines limited to particular cultures. However, an alternative view holds that general cognitive abilities develop as a result of the acquisition of specific skills (e.g., Thorndike and Woodworth 1901; Vygotsky 1962, 1978; Ferguson 1954, 1956, 1959). In this view, cognitive development is best described as the gradual accumulation of a repertoire of cognitive tools and routines that may then transfer to other domains.

The research reported here takes this latter view as its point of departure. This research describes the cognitive basis for a unique skill and then explores the consequences of expertise at this skill for the kinds of general abilities traditionally measured in research on individual differences (e.g., Thurstone 1962). We begin the paper with a brief rationale for our general approach to the study of skills and their development. The remainder of the paper is then devoted to reporting the results of several studies on the acquisition of abacus skill by children in Taiwan. Abacus training is a culturally unique experience that enters the lives of Chinese children to varying degrees. By tracing the cognitive consequences of varied degrees of abacus training, we obtain new insight into the relationships between variations in mastery of particular skills and other skills to which they transfer.

The Development of Skilled Performance

Were we to begin with general cognitive abilities as our unit of analysis, our first challenge would be to define some general ability (e.g., memory or "g") and then devise a method for measuring the ability as an individual trait that is independent of specific performance contexts. We then would proceed to investigate the ways that this general ability is utilized in the acquisition and performance of specific tasks.

A skills approach to the study of cognitive development proceeds quite differently. A skill is a different unit of analysis because it is always defined in terms of specific task content (Fischer 1980). Neisser writes that skill involves both a practitioner and a structural medium. The prototypes of skill are "activities that use certain well-defined objects or manipulate specific physical materials to some end. Playing tennis is such a skill, and so is carpentry" (Neisser 1983, p. 2). Skills involve adaptation both to the medium and to the hierarchy of goals in which the activity is embedded. A comprehensive approach to studying the development of skilled performance necessarily includes three fundamental types of questions, namely, representation, transfer, and context.

Representation.—The first issue that one must investigate in studying the development of skill involves the mental representation of skilled action. This is always studied first in the context of a specific skill. What knowledge is required in order to perform the skill, and how is this knowledge represented mentally? How do these representations change as expertise in the skill is acquired? What are the processes that act on the representations during performance? In the case of the mental abacus, we are interested in knowing in what sense one can claim that abacus experts have an abacus in their heads. In what ways does the mental abacus resemble a real abacus, and in what ways is it different? Or, more broadly, what is it from the environment that is preserved when it is internalized and embedded in a cognitive skill?

Transfer.—Whereas questions of representation are addressed within the context of individual skills, questions of transfer address issues of how different cognitive functions come to be related, or what Vygotsky (1962, 1978) called the problem of "interfunctional relations." How does acquisition of one skill affect acquisition of other skills? What effect does skill in one domain have on understanding or conceptual knowledge, both in that domain and in related domains? How might expertise at a given skill promote the development of more general cognitive abilities that can be applied across a relatively wide range of contexts?

Each of these questions can fruitfully be asked in the context of abacus training in Taiwan. For example, an enduring issue in mathematics education is the relation between calculation skills and problem-solving skills. Although many mathematics educators have called for less emphasis on the rote practice of computational skills, there is virtually no experimental research that assesses the importance that computational skills have in promoting higher-order mathematical reasoning. Abacus training provides us with a natural context for investigating such issues, since intensive training in abacus-based mental computation is carried out quite independently of exposure to other kinds of mathematical training. Does massive practice in mental computation simply lead to faster and more accurate arithmetic calculations, or does it also have implications for children's conceptual understanding of number and general abilities to solve mathematical problems?

Context.—Of course, all skills are acquired within specific social, cultural, institutional, and instructional contexts. In order to understand the development of skilled performance, it first is necessary to understand these contexts. Even if we are primarily interested in the cognitive development of individual children, we must realize that much of that development is a function of processes external to the individual child. For example, abacus training is available to children in Taiwan, but not all children participate. What are the factors that lead some children to participate and others not? Clearly, if abacus training has some impact on cognitive development, factors leading children to spend their time in abacus training may be intimately tied to cognitive development. But these factors do not necessarily reside within the child. Economic factors may lead to differences in opportunity across children, as well as in parental values and expectations regarding child development.

A comprehensive understanding of cognitive development requires research addressing the issues of representation, transfer, and context. Because all three of these issues are involved in the development of any particular skill, findings related to any one may be contingent on the others. For example, the mechanisms of transfer usually reside either in the representation or in the context of skilled performance. One skill may transfer to another because the two skills share a common mental representation. They also may transfer because they co-occur in a particular context, meaning that exposure to the context leads to simultaneous acquisition of both skills. Likewise, mental representations are often affected by the context in which a skill is acquired. The role of motor action in the representation of mental abacus skill, for example, has been shown to vary according to the way in which the skill is taught (Enerson and Stigler 1985).

Overview of the Present Study

The remainder of this paper describes a series of studies conducted in Taiwan, all focused on the acquisition of abacus skill and its consequences. The subjects were elementary school children. Together, these studies illustrate how research on the development of a particular skill may, in a comprehensive way, address questions of representation, transfer, and context. First, what is the nature of abacus skill, and how is it represented mentally? Second, how does acquiring abacus skill affect the child's cognitive functioning in other domains? Third, what is the context in which abacus skill is acquired, and how does this context relate to issues of representation and transfer?

In the first section below, a brief description of abacus training in Taiwan is presented. The skills of abacus and "mental abacus" calculation and the way in which these skills fit into the Chinese cultural context are described. A brief overview of how the abacus works is also provided. Next, previous research on the mental representation of abacus skill is reviewed. Abacus training is shown to have both a quantitative and a qualitative effect on the cognitive processes that Chinese children use in mental calculation.

The final two sections report the results of two new studies that we recently have conducted, addressing questions of transfer and of context. The first study (Study 1) examines the factors that are associated with children choosing to participate in an abacus-training program and the consequences of this training for their future performance in school. The second study (Study 2) goes beyond the functional relationships discovered in Study 1 to examine mechanisms that may account for the functional consequences of abacus skill. We end by considering how one might move from studies of specific skills to an understanding of more general factors that lead to the development of expertise.

Abacus Training in Taiwan

Computational procedures are basic skills that are used across a wide variety of cultures. How people learn and represent these skills, however, differs greatly as a function of the cultural context in which these skills are learned and applied. In the United States, for example, complex or rapid mental computation is not considered a particularly important skill, nor is the skill routinely taught in American schools. Expertise in mental calculation is acquired only by a small number of individuals, using idiosyncratic techniques and working independently (Smith 1983).

Asian cultures provide a strong contrast. In Japan and Taiwan, for example, one finds efforts to train children in mental computation, both in school and in extracurricular programs. An emphasis on speeded mental calculation is included in the regular school curriculum beginning in the first grade. Stigler, Lee, and Stevenson (1986) describe a first-grade Chinese mathematics class in which 20 separate activity segments in a single 40-minute period are devoted to drill aimed at speeding up children's already correct retrieval of the simple addition facts. Stigler et al. report that such classes are not atypical among Chinese classrooms. As children enter the older grades, they are drilled on more complex mental calculations, such as multiplication of two two-digit numbers. In all cases, speed is considered to be essential to success in mathematics. Cognitive theories that stress automaticity (e.g., Shiffrin and Dumais 1981) fit well with Asian folk theories about the development of cognitive skills.

Abacus training can best be understood within this broader context where rapid mental calculation skills are highly valued. Children in Taiwan are first introduced to the abacus as part of the fourth-grade elementary school mathematics curriculum. (Training in conventional paper-and-pencil methods of computation begins in first grade, as it does in the United States.) All children are taught the basic principles of addition, subtraction, multiplication, and division with an abacus. However, the training provided in school is not sufficient for developing a high level of skill. Children who wish to become experts in abacus and mental abacus calculation undertake more intensive training by enrolling in programs that meet after school, called *buxiban*. These after-school programs are usually associated with an elementary school, and the participants engage in interschool competitions involving both abacus and mental computations. They also take certifying exams, administered by the government-run Chinese Abacus Association, which rate participants according to their expertise in abacus and mental computation.

Instruction in mental abacus calculation proceeds as follows. Children are instructed to visualize a mental image of the abacus and then to manipulate the "beads" on this mental abacus following the same procedures that they would use on a physical abacus. Beginners at mental abacus calculation move their fingers in the air to aid in moving the mentally imaged beads. Hatano et al. (1977) have shown that interfering with these finger movements interferes with the speed and accuracy of mental abacus calculations. Experts are able to perform mental abacus calculations without moving their fingers.

Children acquire abacus skill both in school and in specialized after-school training programs. Because the present research was carried out at Dongyuan Elementary School in Taipei, Taiwan, abacus training in this particular setting will be described. Dongyuan is a large urban school with about 4,500 students in grades 1–6. The school itself is typical of public schools in Taipei, but it has an unusually good abacus program (according to the commissioner of Taipei schools).

Complementing the abacus program at the school is Dongyuan Buxiban, an after-school program directed by the elementary school's abacus teacher. Dongyuan Buxiban is devoted solely to abacus instruction for children from Dongyuan Elementary School. Children who want to pursue additional abacus training can attend classes three afternoons per week at the *buxiban* for one to 1½ hours each afternoon. Of the approximately 750 students at each of grades 4–6, about 100 from each grade elect to attend these abacus classes after school. A small number of students in first, second, and third grades also attend classes at the *buxiban*. Members of the abacus team that will represent Dongyuan Elementary School in competition are chosen from among those children who attend classes at the *buxiban*. Anyone is welcome to join the *buxiban* program, and a large cross section of children attend.

From the American point of view, the goings-on at Dongyuan Buxiban are quite impressive. Children go there after school and usually sit on long benches in large rooms filled with students. A typical mental abacus calculation exercise begins when the teacher, standing at the front of the room, raises his hand, whereupon the room falls silent in anticipation. The teacher then reads aloud a list of 20 three-digit numbers as fast as he can, so fast in fact that the numbers are almost unintelligible. The children are silent, and the room is tense with concentration. After the last number is read, every hand in the room shoots up, and the teacher calls on one child to report the sum. Usually the child's answer is correct.

The Abacus and Abacus Addition

Before presenting the research on consequences of abacus skill, it is helpful to briefly describe the abacus and abacus calculation. (Readers familiar with abacus operation may skip this section.) The abacus most widely used today, and thus the subject of the present study, is the Japanese abacus, or *soroban*. It is a modern adaptation of the Chinese abacus and took its present form around 1920 (Kojima 1954*b*). The *soroban* is a wooden, framed instrument made up of 23 columns of beads, as illustrated in figure 1.

The abacus uses place value and the base 10 system of numeration to represent numbers. Each column of beads has a place value, corresponding to the ones, tens, hundreds, thousands, and so on. Which *specific* column of beads is defined as the "ones" is a matter of choice and convenience, but once it has been defined, all other columns will be valued relative to it. The columns on an abacus thus correspond to the columns of a base 10 numeral: 1,452, for example, is represented on four adjoining columns of abacus beads.

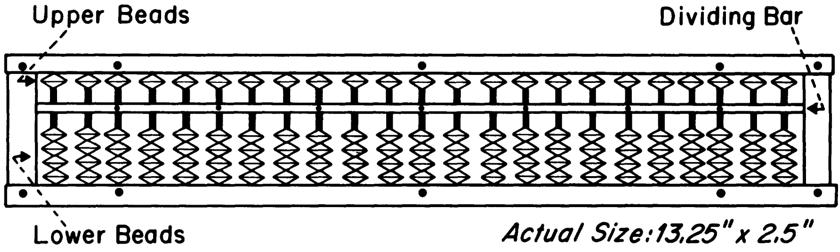

FIG. 1.—The Japanese abacus, or *soroban*

As can be observed in figure 1, each column of beads is divided into an upper and a lower section. The bead in the upper section is equal to five times the units value of the column (5, 50, 500, etc., depending on the place value of the column) when it is pushed down toward the dividing bar that separates the two sections. If it is away from the dividing bar, then it is equal to zero. Each of the four lower beads is equal to one time the units value of the column (1, 10, 100, etc.) when pushed up toward the dividing bar, and zero when pushed away from the bar.

By pushing different combinations of beads toward the dividing bar, it is possible to represent the numbers zero through nine on any single column. For example, if the upper bead were pushed down and the four lower beads all pushed up, the number nine would be represented. If the upper bead were pushed back up, but the lower beads left in place, the number four would be represented.

The basic units of abacus addition are the single-digit additions. For each single-digit number zero through nine that could be represented on a single abacus column, one could add any of the single-digit numbers one through nine. There thus are 90 different possible single-digit additions, and each is accomplished with a specific stereotypical finger movement.

Multiple-digit addition is accomplished simply by executing sequences of the single-digit additions. Numbers are added a digit at a time, moving from left to right through each successive addend. For example, the addition of 123 + 456 proceeds as follows: first the one is set on the abacus, followed by the two and then the three to yield the first addend. Then the next addend is processed from left to right: first the four is added to the one; then the five is added to the two; and finally the six is added to the three. At this point the answer, 579, appears on the abacus. If there were a third addend, it would be processed from left to right also, and added into the 579.

As is apparent from the preceding example, it is possible to take any addition problem and generate the sequence of single-digit additions that would be used to solve it on the abacus. Thus, the problem 123 + 456 would generate this sequence: 0 + 1, 0 + 2, 0 + 3, 1 + 4, 2 + 5, 3 + 6. One could also generate a list of the intermediate states the abacus passes through in the solution of a problem, describing states with the number represented on the abacus after each of the additions in the sequence. Again in the case of 123 + 456, the abacus would pass through the following states: 000, 100, 120, 123, 523, 573, 579. (For further information about the abacus and its use, refer to Kojima [1954*a*, 1954*b*]; Moon [1971]; Stigler [1982]; or Tani [1964].)

Previous Research: Stalking the "Mental Abacus"

Our initial studies of abacus skill focused on the "mental abacus." The phenomenon of mental abacus calculation seemed particularly interesting because it was a case in which a culturally specific activity employing a culturally specific technology appeared to have both a quantitative and qualitative effect on the cognitive processes of individuals. The research had two major goals: (1) we wanted to document the calculating speed and accuracy of experts using a mental abacus; and (2) we wanted to understand the nature of the mental abacus, that is, how it is represented mentally and how it functions. We will briefly describe some of the results of these studies. A full report can be found in Stigler (1984).

The subjects for these studies were 12 fifth-grade students from Dongyuan Elementary School. They ranged in age from 10 years, nine months, to 11 years, six months. All were considered good students, with average school grades (on a 100-point scale) ranging from 85 to 95. The students were selected in consultation with the abacus teacher to represent three levels of expertise in abacus skill: four experts, four intermediates, and four novices. The four experts had received the highest possible rating (*duan wei*) from the Chinese Abacus Association (Zhusuan xuehui 1980). The intermediates were rated as Grade 5 (a somewhat lower rating), and the novices had no rating. Experts and intermediates had been attending the after-school abacus program at Dongyuan Buxiban, while the novices had received only the training provided during the regular school day.

Calculation Speed

Subjects were presented with 256 addition problems, ranging in complexity from two addends of two digits to five addends of five digits. Sixteen problems were constructed for each of the 16 cells of the matrix defined by two, three, four, and five digits per addend crossed with two, three, four, and five addends. Experts and intermediates solved each problem twice—once mentally and once using the abacus. Since novices had never been trained in abacus-based mental calculation, the problems were too difficult and time consuming when solved mentally. Novices, therefore, were asked only to perform the abacus calculations.

The computation times for experts, intermediates, and novices broken down according to problem type are displayed in figure 2. Most interesting are the times that experts took to complete these sums. One can see in the right-hand panel of figure 2, for example, that expert abacus operators mentally were able to add five five-digit numbers in about seven seconds. For experts, abacus calculation was slower in general than was their mental calculation, whereas for intermediates

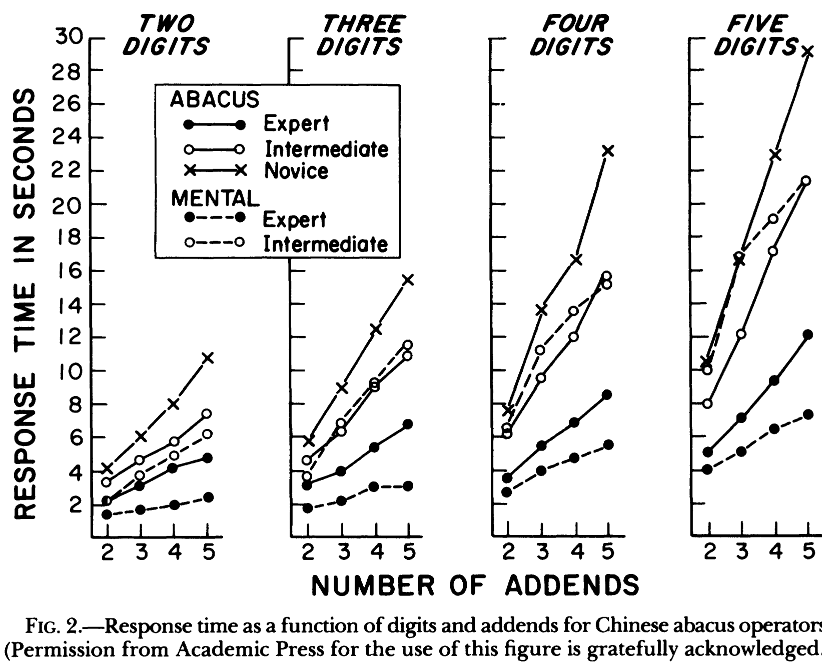

FIG. 2.—Response time as a function of digits and addends for Chinese abacus operators. (Permission from Academic Press for the use of this figure is gratefully acknowledged.)

the two modes of calculation were accomplished at approximately the same speed.

What's Abacus-like about the Mental Abacus?

We have demonstrated the astounding mental calculation speed of the young abacus experts. However, there are many routes by which one can achieve extraordinary speed in mental calculation (see, e.g., Smith 1983). Practice with any skill leads to improvement, and these children certainly had been practicing mental calculation a great deal. In what sense did the abacus have an impact on the children's cognitive processing, over and above the simple quantitative effect of mental calculation practice? What does it mean to have an abacus in your head? In other words, what aspects of the environment are preserved when incorporated into the individual's cognitive processes?

Our previous research shows that the mental representations used by abacus experts in carrying out mental calculation reflect the structure and functioning of the physical abacus. Our general research strategy has been to use the abacus itself as a hypothetical model of what goes on in mental abacus calculation. Chinese abacus experts' mental calculations have been compared with their abacus calculations. Both of these, in turn, have been compared with American students' mental calculations, which obviously are not based on abacus training. The highlights of this research are presented next.

Access to intermediate states.—One piece of evidence for the existence of the mental abacus came from the Intermediate States Task. As described earlier, the abacus goes through a sequence of intermediate states en route to the solution to any problem. While we know that abacus experts can quickly report the final state in the solution process—that is, the answer—we reasoned that if in fact they were manipulating a mental abacus just as they would an actual abacus, they should also be able to answer questions about intermediate states unique to the abacus solution of the problem.

The Intermediate States Task was designed to test this hypothesis. In this task, subjects were presented with a series of four-addend three-digit addition problems and a probe card containing a photograph of a particular abacus configuration. Sometimes the probe card contained an actual intermediate state in the solution of the problem, and other times it did not. Each subject was asked to say as quickly as possible whether or not the probe contained a valid intermediate state in the solution of the problem. Subjects performed the task both mentally and using an abacus.

The results of this study supported the existence of a mental abacus. Both experts and intermediates at abacus calculation were able to manipulate a mental representation of the abacus so that it passed through the same states that an actual abacus would pass through in solving an addition problem. This was evidenced in several ways. First, subjects were able to answer correctly—and without using an actual abacus—questions about intermediate states unique to the abacus-based solution of a problem. Second, response times for answering such questions proved to be a function of the position of the probed state in the sequence required for solving the problem (i.e., the further along in the sequence, the longer it took subjects to respond to the probe). Finally, the pattern of response times did not differ whether the task was being solved mentally or using the abacus.

Analyses of errors.—Perhaps the most compelling evidence for the incorporation of specific abacus-like characteristics into the abacus expert's mental representations came from the analyses of errors made by both abacus experts and American students. (The American sample in this case comprised four university students. American fifth graders were unable to complete the mental calculations required for comparison.) Various error analyses were conducted by Stigler (1984). One illustrative example will be presented here.

As described earlier, the abacus is divided into upper and lower sections. The lower beads each represent one and the upper bead represents five. We hypothesized that, given this property of the abacus, one might be more likely, all other things being equal, to make errors that are off by exactly five when using an abacus than when using some other method of calculation.

The hypothesis was supported strongly. When using an abacus, 22.6 percent of the calculation errors produced by the abacus operators in our sample were off in at least one column by exactly five. When the same subjects were engaged in mental calculation, the percentage of errors off by five was 20.4 percent. American students, by contrast, produced only 3.7 percent errors of five while engaged in mental calculation. Clearly, a fundamental structural characteristic of the abacus was preserved in the Chinese children's mental representation of the abacus.

We have documented both the quantitative and qualitative impact that abacus training has on Chinese children's mental calculation skill. We turn now to two studies aimed at addressing questions of context and of transfer. In doing so, we move our focus from the performance of individual children on a single task to a broad investigation of the relationship of abacus skill to other skills, both within and across Chinese children.

Study 1: Functional Consequences of Abacus Skill

The previous studies examined the mental representation of abacus skill and explored the nature of the "mental abacus." The four 11-year-old abacus experts we studied had attained remarkable skills in mental calculation. Were American children to demonstrate such skills, they might be considered prodigies. Yet within the Chinese context, there is nothing surprising about these skills. Abacus teachers claim, in fact, that anyone who invests the requisite time can attain comparable expertise.

These claims lead naturally to the questions that motivated the next study. One goal of the next study was to place abacus training in its cultural and educational context. Who are the children who elect to participate in extracurricular abacus training, and how do they differ from other children? Of those who do participate, what factors are associated with becoming expert? Is it true that anyone who practices can master the mental abacus, or must they be special in some way prior to training?

In order to answer these questions, we had to hypothesize the kinds of factors that might be associated with abacus training and with the acquisition of expertise. For this first study, we narrowed the field to four kinds of factors. These were the child's (1) family background in terms of socioeconomic status; (2) previous level of general intellectual ability; (3) previous level of achievement in school, both in mathematics and in other subjects; and (4) history of participation in abacus training. In Western education contexts, socioeconomic status is known to have a pervasive relationship to children's learning (Musgrave 1972), so we decided that it should be included in this study. The other three factors—roughly equivalent to ability, knowledge, and practice—are those factors most often theoretically associated with learning.

Another important goal of this study was to examine the consequences of abacus skill for cognitive functioning in other domains. In this preliminary study, the issue of transfer was addressed by examining the effects of abacus training on subsequent performance in school (as evidenced by grades in mathematics and reading). Skill in mental abacus obviously implies skill in rapid mental calculation. But do these calculation skills have any impact on children's overall performance in elementary school mathematics? If there is a positive effect, is it specific to mathematics or does it also extend to academic performance in general, as evidenced by performance in another subject such as reading?

We were interested not only in transfer to performance domains but also in the relationship between abacus training and the development of self-concept. Self-concept and achievement are typically found to be related, although the direction and mediators of that relationship are not always apparent (Hansen and Maynard 1973). Experimental work shows that improved self-concept is an outcome but rarely a cause of successful academic performance (Scheirer and Kraut 1979). There is complementary evidence that extracurricular skill development can enhance children's self-concept (Koocher 1971). Does abacus skill affect students' conceptions of their academic competence? If so, how is that effect mediated by academic performance?

In summary, we first examined selection factors that determine which children enroll in the after-school *buxiban*. We next examined factors that determine the level of expertise that is achieved by those students who do enroll in the program. We also examined the effect of abacus skill on children's performance in school, specifically in mathematics and reading. Finally, we examined the effect that abacus training has on children's self-perceptions of cognitive competence.

Method

Whereas the previous studies involved intensive investigation of a small number of children, the present study required investigation of large samples of children over time. This study made extensive use of school records because these records provided valuable background data on each child.

Subjects.—The study was conducted at Dongyuan Elementary School, the same school in which the previous research was conducted. From the entire cohort of 714 fifth-grade students, complete data were available for 618 children. Of these, 327 were male and 291 female. The average age was 11.3 years (SD = 0.35). These students were evenly distributed across all 16 fifth-grade classrooms. Subjects not included because of incomplete data typically were students who had not been in the school since first grade and who therefore had incomplete records. When compared with students who were included in the study, they were not found to differ significantly ($P > .05$) on any study variable for which the two samples could be compared.

Approximately 15 percent of the fifth-grade students had been attending the after-school abacus-training program; complete data were obtainable on 97 such students. These students had begun their extracurricular abacus training some time during their second-, third-, fourth-, or fifth-grade year in school. No students had begun abacus training in first grade.

Measures.—Three indicators of socioeconomic status (SES) were obtained: the mother's educational attainment, the father's educational attainment, and the status of the father's occupation. Educational attainment was measured in total number of years of formal schooling, and occupational status was coded on a five-point scale. A summary variable was constructed by taking the first principal component of the three measures. This component, which was the only one with an eigenvalue exceeding unity, accounted for 66 percent of the variance in the original three measures.

General cognitive ability was measured in the first grade by scores on Raven's Colored Progressive Matrices, a standardized test administered to all children midway through the first grade. The Raven's scale has been used extensively in Taiwan and has been shown to be both valid and reliable in the Chinese context (Hsu 1971; Hsu, See, and Lin 1973).

First-grade reading and mathematics grades were used as measures of each child's academic achievement prior to abacus training. Reading and mathematics grades from the end of fifth grade were used as measures of academic achievement after exposure to abacus training. Grades are measured on a 100-point scale and are based on schoolwide tests administered monthly. Therefore, semester grades in Taiwanese elementary schools are substantially more objective than are grades in most American elementary schools.[1]

Each child's history of participation in the abacus-training program and a rating of how expert each child had become were provided by the head abacus teacher and director of the abacus-training program. This information was collapsed into two summary variables: (1) the total number of hours spent in organized after-school abacus practice through the end of fifth grade (HOURS), calculated from records of students' *buxiban* attendance, and (2) the degree of skill acquired (SKILL), rated along the 10-point scale given in formal certification requirements of the Chinese Abacus Association. This latter variable was computed as the average of two separate ratings, one for mental and one for physical abacus skill. The two ratings were highly correlated in this sample ($r = .92$), as our previous research had lead us to expect.

Each student was administered the Perceived Competence Scale for Children (Harter 1982). The cognitive subscale, which measures children's self-ratings of their performance in school-type tasks, was used as an indicator of perceived cognitive competence. Stigler, Smith, and Mao (1985) have shown that the structure of this scale is valid for a Taiwanese sample.

Analyses.—The data were subjected to two multivariate analyses. First, a stepwise discriminant analysis was performed on the entire sample to determine if students who volunteered for abacus training differed significantly from students who did not volunteer on any of the variables measured. Next, to determine precursors to and consequences of abacus skill, a path analysis was performed on the data for students who had taken abacus training.

Path analysis was accomplished using methods described by Joreskog and Sorbom (1983). These methods allow the investigator to hypothesize a set of causal relationships that obtain among a set of variables and then to test the covariance matrix to determine if the observed data are consistent with the hypothesized model. A poor fit of the path model to the data indicates that the proposed causal model does not obtain. A good fit of the model supports the validity of the model, though it does not rule out plausible alternative models that have not been tested.

In practical terms, a hypothetical path model is proposed and then is fit to the observed covariance matrix by the LISREL computer program. The program estimates the standardized path coefficents (comparable to standardized Beta weights in multiple regression). It then provides a chi-square test of the overall fit of the model. Other statistics, such as *t*-values for each path, provide a test of the significance of individual paths. Modification indices are provided to help the user determine which missing paths would improve the model's fit were they to be included. These various pieces of statistical information then can be used to construct an improved model.

Results

Who gets abacus training?—The Raven's Matrices score, SES score, mathematics grades from first grade, reading grades from first grade, and child's sex were entered into a stepwise discriminant analysis using forward selection of variables. The objective of this analysis was to build a "best" model to predict whether a student would or would not choose extracurricular abacus training. The significance level for entry was $P < .1$; the significance level for retention in the model was $P < .12$. The liberal entry and retention values were chosen to minimize the Type-II error rate.

At the first step, all five variables met the minimum criterion for entry. In other words, at the level of separate univariate tests, those who chose training had, on average, higher first-grade Raven's scores, came from families of higher SES, achieved higher mathematics and reading grades in first grade, and were more likely to be male. The best discrimination was provided by first-grade mathematics grades, although the R^2 was quite low ($R^2 = .03$, $P < .001$). After first-grade mathematics grades had been entered into the model, only sex met the criterion for entry at the second step (partial $R^2 = .005$, $P = .07$).

After first-grade mathematics grades and sex had been added to the discriminant function, no further variables met the entry criterion. It was concluded that students with higher first-grade mathematics grades and males were more likely to elect abacus training. However, as can be surmised from the poor R^2 statistics resulting from these variables, their contribution was of no practical value. Our best model could not categorize any individual child into the abacus-trained group. Examination of the trained and untrained groups' distributions showed that this was due to their almost complete overlap. In other words, the average student choosing extracurricular abacus training was different from the average student not choosing the training, but differences were slight.

What are the causes and consequences of skill?—The next analysis involved only those students who had participated in the abacus-training program. Given that they all had elected to receive abacus training, what factors determined how skillful, or expert, each would become? Further, what impact did level of abacus skill attained have on subsequent grades in school and on perceived competence?

We began by hypothesizing a model on the basis both of relationships reported in the literature and of our own intuitions. Our initial model is diagrammed in figure 3. Paths that ultimately were retained are given in boldface; paths that proved not to be significant ($P > .05$) and that were deleted from the final model are shown in lighter print. Variables depicted at the left were measured in first grade, those at the right in fifth grade, and those in the middle were assumed to apply sometime during the five-year period.

Examination of figure 3 shows that we expected SES to have an impact on both mathematics and reading grades at both the first and fifth grades, as well as on the Raven's score and perceived cognitive competence. At first grade, we expected that reading (READ), mathematics (MATH), and Raven's all would be intercorrelated, though there was no causal direction implied. We expected abacus skill (SKILL) to be a function of previous ability (RAVENS), previous school achievement (READ and MATH), and number of hours spent practicing the abacus. Fifth-grade mathematics and reading grades were expected to be intercorrelated and to be largely a function of previous reading and mathematics grades. Abacus skill was hypothesized to have consequences for reading, mathematics, and perceived cognitive com-

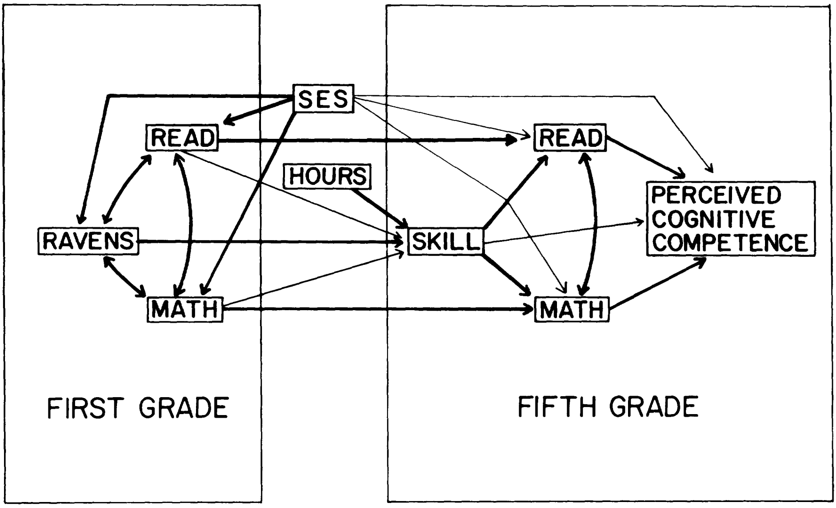

FIG. 3.—The hypothesized path model for Study 1

petence. Perceived cognitive competence was expected to result from actual competence as measured by grades in school, as well as from abacus skill.

The overall fit of this model was evaluated by the residual χ^2 test provided by the LISREL program.[2] The residual χ^2 test, which LISREL provides instead of the similar R^2 index, is a test of the significance of variance unexplained by the model. If a model fits well, the residuals should not be statistically different from zero. Thus a model that fits the data yields an insignificant residual χ^2. Our initial model fit the data only marginally: χ^2 (15) = 22.97, P = .066.

We proceeded to improve the model in the following way. The t-values for individual paths were used to locate paths that were superfluous. Insignificant paths ($P > .05$) were deleted. Modification indices were used as described in the manual to locate paths that had not been specified but that were required to improve the model's fit. Finally, four pairs of paths that appeared equal and whose equality was theoretically plausible were constrained to be equal, causing no detriment to the fit of the model.[3]

The final model, including standardized path coefficients, is presented in figure 4. This model fits the data quite well: $\chi^2(24)$ = 23.96, P = .464. Several features of the resulting model are worth stressing. Abacus skill is found to result primarily from practice. Reading and mathematics achievement at first grade have no effect on subsequent abacus skill,

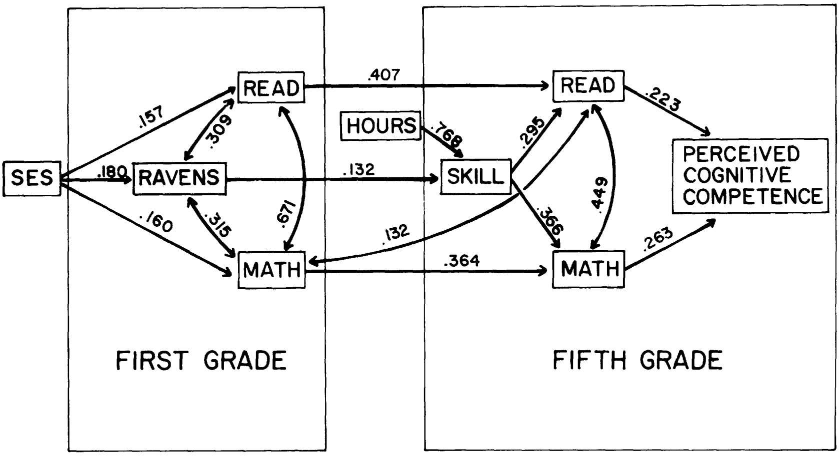

FIG. 4.—The final model fit for Study 1

and the impact of Raven's Matrices performance on abacus skill is small relative to the practice effect.

Socioeconomic status exerts its influence only at first grade, having direct effects on reading, mathematics, and performance on Raven's Matrices. The impact of SES in later years is not direct but is a carryover from its influence at the beginning of schooling. Perceived cognitive competence in fifth grade is equally affected by concurrent reading and mathematics performance. Abacus skill has no direct effect on perceived cognitive competence, but abacus skill does have an indirect effect via its impact on reading and mathematics performance. Abacus skill has a positive effect on subsequent mathematics and reading grades, though the impact on mathematics grades is larger than on reading grades.

Discussion

These results demonstrate that abacus skill is primarily a consequence of practice and that the eventual impact of abacus skill reaches into other domains. The functional relationship between abacus training and improved academic performance in reading and mathematics indicates that transfer has occurred. However, it tells us little about the mechanisms of transfer, of which there could be many.

Some possible mechanisms may be predominantly contextual in nature, whereas others may be more centered on cognitive characteristics of the child. A contextually based mechanism for transfer, for example, might be that teachers favored students who went to abacus class by giving those students higher grades. Two observations detract from the plausibility of explaining the present effects in terms of teacher favoritism. First, Taiwanese teachers grade students on the basis of performance on schoolwide tests. This grading procedure leaves little room for addition of systematic increments to abacus students' grades. Second, the effects of abacus skill are systematic. Teacher favoritism might raise grades for students who participate in the after-school program, but there is no reason to expect grades to be raised more for experts than for other participants.

A cognitively based hypothesis for the mechanism of transfer is that underlying ability factors (e.g., spatial ability) were enhanced, and that these, in turn, aided academic performance. Previous attempts to describe skill transfer in terms of Ferguson's (1954, 1956, 1959) theory have assumed that skill transfer is due to changes in fundamental ability factors (e.g., Buss 1973). According to this view, the structure and/or level of fundamental abilities change as new skills are incorporated. As a result, new skills are built from a base of more abstract, more complex, and/or more appropriate abilities. These improvements in fundamental abilities are thought to account for apparent transfer between skills.

This latent-ability model is more complex than would be required to explain the transfer from abacus skill to mathematics achievement. On the other hand, it might provide an explanation for the transfer to reading. For example, tasks similar to abacus have been found to be effective in training basic spatial ability, and training in spatial tasks has been shown to enhance reading performance (Johnson and Crano 1977). Perhaps abacus training transfers to reading via an effect on spatial ability.

One final potential explanation for transfer from abacus skill to other domains states that transfer can be accounted for by the effect of abacus skill on component subskills that are shared across domains (see Sternberg 1985). For example, it may be that abacus skill enhances students' ability to calculate and that this ability, in turn, is fundamental to mathematics achievement at the grade levels that we are studying. This could account for the transfer from abacus skill to mathematics achievement, but it is unclear how such a theory would explain the effect on reading achievement.

In summary, after eliminating teacher bias and self-concept as likely mediators of transfer, we have outlined two alternative cognitive explanations. These are not necessarily exhaustive. They are suggested here because they are the ones that we found most plausible in constructing our next study.

It should be pointed out that the explanations are not mutually exclusive. As noted earlier, there already is convincing evidence that an analog mental representation of abacus skill results from abacus training and that this mental representation facilitates mental calculation (Stigler 1984). This may enhance performance on paper-and-pencil mathematics tasks that, in turn, contribute to achievement in mathematics courses. However, the basis for transfer to reading may differ from that for transfer to mathematics. While transfer to mathematics may revolve around improvement of component subskills, transfer to reading may involve improvement of spatial skills that in turn enhance reading achievement (see Johnson and Crano 1977). These hypotheses are tested in the subsequent study.

Study 2: Searching for Mechanisms

We have seen that abacus skill primarily is a function of practice and that it has a positive effect over time on both mathematics and reading grades in school. But this functional relationship tells us little about the mechanisms that underlie the consequences of abacus skill. We know that abacus training improves calculation skills. For example, if grades in Taiwan are based primarily on calculation, the finding that abacus skill improves mathematics grades is trivial.

The purpose of this second study was to gather additional information that might shed light on the mechanisms by which abacus training exerts its impact on performance in other domains. We returned to Dongyuan Elementary School one year later, when the sample of fifth graders from Study 1 was nearing completion of sixth grade. Students were given a new battery of tests. This battery was chosen to test the alternative explanations that we had elaborated for results from Study 1. A more complete picture emerged.

Method

Subjects.—Complete data were available for 81 of the 97 abacus trainees tested the year before. Forty-seven of the students were male, and 34 female. No significant differences were found on any prior course grades or test scores between students who were followed up and those who were not followed up.

Measures.—Level of abacus skill (SKILL) and reading and mathematics grades were available as in Study 1 and were used in the present study.

Social studies grades were added to assess whether abacus skill would transfer to a different academic domain as it had to reading and math. In addition, each child was given a battery of tests consisting of the following:

1. Mental calculation tests. Since we knew that mental calculation skills are a primary consequence of abacus training, we wanted to measure these skills directly and not simply to infer them from abacus skill ratings. Two tests were given: one in mental addition, the other in mental multiplication. Each addition problem required the student to add five three-digit numbers. Each multiplication problem required the student to multiply two two-digit numbers. Problems on both tests were typed on sheets of paper, and each student was provided with many more problems than could be solved within the time limit. Students were told to solve each problem mentally and then write down the answer. They were given 90 seconds to do each test and were told to work as quickly as possible. The resultant variables—ADD and MULTIPLY—were scored as the number of problems solved correctly within the time limit.

2. Stanford Diagnostic Mathematics Test (SDMT). We knew that mathematics grades were positively affected by abacus skill, but we did not know what specific *kinds* of mathematical skills were improved by abacus training. To investigate, we developed a Chinese translation of the SDMT (Beatty, Madden, and Gardner 1978). The SDMT is a group-administered test of mathematics achievement that yields a summary assessment in each of three areas: calculation, applications, and number system and numeration.

The calculation subtest includes paper-and-pencil calculation problems covering addition, subtraction, multiplication, and division of whole numbers, decimals, and fractions. The applications subtest comprises a standard set of arithmetic word problems. The numeration subtest is designed to test conceptual knowledge about numbers and the number system. It places heavy emphasis on children's understanding of place value and on their comprehension of large numbers.

The version of the test that would be appropriate for sixth-grade students in the United States was used. However, it proved somewhat easy for our sample, leading to less than optimal distributions. The translation had been checked with Chinese teachers and educators, who were satisfied that the test was not culturally biased against Chinese students.

3. Primary Mental Abilities Test. Two subscales from the Primary Mental Abilities Test (Thurstone 1962) were administered. These were the number series test (SERIES) and the spatial relations test (SPACE). We chose these tests for specific reasons.

We were interested in explaining the observed relation between abacus skill and reading achievement. Spatial ability has been hypothesized to be a link between these two skills (Johnson and Crano 1977). Therefore, the spatial relations subscale was included here so that we could obtain a direct measure of this hypothesized link.

We expected that performance on number series problems would be facilitated by rapid mental calculation skills because the solution of series problems depends, in part, on the ability to formulate and test, via calculation, hypotheses about the rule used to generate the series. We reasoned that the easier it is for a child to calculate mentally, the more likely the child would be to test alternative hypotheses. Therefore, this subtest was included to tap a possible consequence of abacus skill.

4. Attitudes toward school. We speculated that abacus training could improve a child's attitude toward mathematics. In fact, one could construct scenarios in which improvement in attitude actually causes the subsequent improvement in mathematical skills (see Bar-Tal and Saxe 1978). To assess attitudinal consequences of abacus training, each child was asked to rate how much he or she liked reading, mathematics, art, science, and social studies. Ratings were made on a seven-point scale with end points labeled "Dislike it very much" and "Like it very much."

Results

Zero-order correlations between abacus skill and the newly included variables are given in table 1. Examination of table 1 shows that abacus skill has small but statistically significant correlations with grades in mathematics and reading, with all three parts of the SDMT, with spatial ability, and with liking for mathematics. As expected, the highest correlations are found with mental arithmetic ability. Abacus skill does not correlate significantly with SERIES, social studies grades, or liking courses other than mathematics.

In constructing our model, we sought to learn whether the correlation of SKILL with the SDMT subscales, with mathematics grades, and with liking for mathematics could be explained by the previously demonstrated effect of abacus skill on mental calculation (Stigler 1984). We also wanted to know if the correlation between SKILL and reading grades might be mediated by an effect of SKILL on spatial ability (Johnson and Crano 1977). Thus we began our analysis with the model shown in figure 5. As before, paths that were eventually retained are shown in boldface, and paths that were ultimately eliminated are shown in lighter print.[4]

TABLE 1

Zero-Order Correlations of Abacus Skill with Other Variables

Variable	Pearson *r*
Grades in school:	
Mathematics (MATH)	.308**
Reading (READ)	.343**
Social studies (SOCIAL)	.181
Stanford Diagnostic Mathematics Test:	
CALCULATION	.281**
NUMERATION	.363***
APPLICATION	.227*
Mental Calculation Test:	
ADD	.379***
MULTIPLY	.637***
Primary Mental Abilities Test:	
SERIES	.091
SPACE	.231*
How much do you like . . . ?:	
Math	.326**
Reading	.084
Science	.014
Social studies	.117

* $P < .05$.
** $P < .01$.
*** $P < .001$.

Examination of figure 5 shows that we began with the hypothesis that abacus SKILL has a direct effect on mental calculation skills and that mental calculation, in turn, leads to improvement in numeration, calculation, applications, series, and spatial relations. These variables, in turn, are expected to explain improvement in mathematics achievement. Liking for mathematics is expected to result from mathematics achievement.

In order to account for effects due to prior context, first-grade Raven's Matrices score, SES, and first-grade grades in reading and mathematics were partialed out. The *t*-ratios in sequential model fits were then used to eliminate insignificant ($P > .05$) paths. Once all insignificant paths had been eliminated, *F*-ratio tests were conducted to determine if there were significant direct effects not accounted for

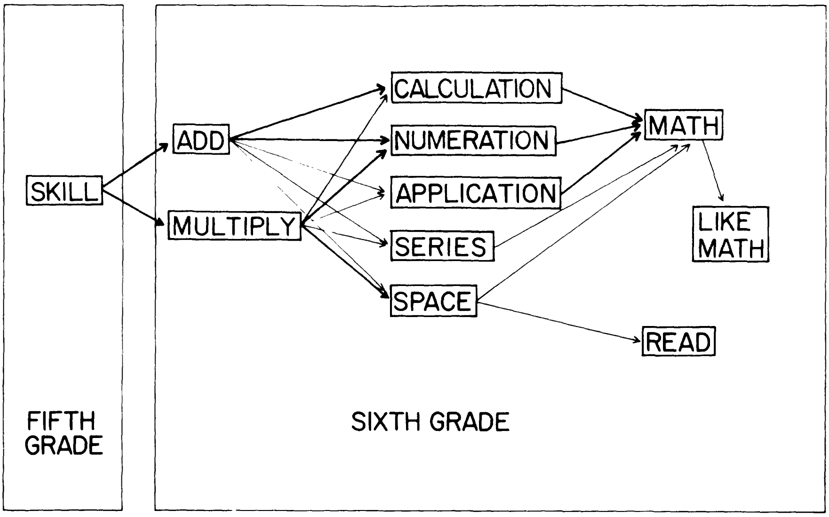

FIG. 5.—The hypothesized path model for Study 2

by the initial model. Finally, as before, path equalities were tested where appropriate.

The *t*-ratios for paths of sequential fits confirmed the effect of skill on both mental calculation variables, ADD and MULTIPLY ($P < .001$). Mental addition was found to positively effect CALCULATION ($P < .001$), while mental multiplication was found to effect NUMERATION ($P < .005$), CALCULATION ($P < .001$), and SPACE ($P < .03$). No effect of either mental calculation variable was found on APPLICATION or SERIES ($P > .05$). The variables NUMERATION, CALCULATION, and APPLICATION were found to have direct effects on mathematics grades ($P < .001$), but SPACE and SERIES were not ($P > .05$). In addition, SPACE did not have the expected impact on reading achievement ($P > .05$).

Subsequent *F*-ratio testing showed that liking for mathematics was significantly affected by NUMERATION and CALCULATION ($P < .02$). Once these effects were entered, the effect of mathematics grades on liking for mathematics dropped ($P > .05$). Significant effects ($P < .05$) were found from NUMERATION onto APPLICATION, and from SPACE onto SERIES.

Using procedures similar to those used in Study 1, three pairs of path equalities were located and then constrained as equal: ADD and MULTIPLY were found to have equal effects on CALCULATION;

NUMERATION and CALCULATION were found to have equal effects on mathematics grades and to equally affect liking for mathematics.

The resultant model, along with the standardized path coefficients, is presented in figure 6. In contrast with the LISREL analysis used in Study 1, the R^2 test provided by the present analysis measures the total amount of variance accounted for by the model. A significant R^2 indicates that the model accounts for a significant proportion of the variance. The final model shows moderately good fit to the data ($R^2 = .333, P < .001$). Several points about figure 6 are worth stressing. First, the effect of abacus SKILL on mathematics grades is explained by SKILL's effect on mental calculation (ADD and MULTIPLY) and by the subsequent effects of mental calculation on arithmetic ability (CALCULATION, NUMERATION, and APPLICATION). Mental multiplication has a small impact on SPACE, which, in turn, has a small impact on SERIES. However, these abilities do not affect mathematics grades. The anticipated effect of spatial ability on reading achievement was not found. The impact of abacus skill on reading achievement thus remains unexplained.

It is also of some interest that the relation between liking for mathematics and mathematics grades is explained by their shared variance with NUMERATION and CALCULATION. Liking for mathematics is apparently a function of facility with basic skills rather than of

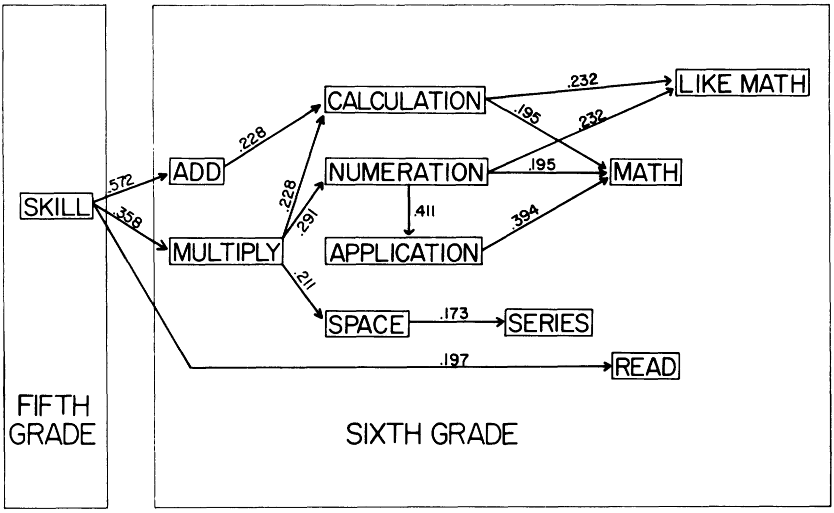

FIG. 6.—The model fit for Study 2

reinforcement via grades. In this context, it is interesting to note that the correlation between reading grades and liking for reading is insignificant: $r(79) = .058$, $P > .5$.

Discussion

In our second study, two cognitive explanations of transfer were combined into a single structural model. The effect of abacus training on mental representation and the beneficial consequences of that representation for mental calculation had been demonstrated previously (Stigler 1984). This study confirmed those findings and elaborated their further consequences. Abacus skill enhances mental calculation, which, in turn, transfers to component mathematical subskills of numeration and calculation. These subskills equally affect mathematics achievement. The transfer from abacus skill to numeration has a further indirect impact on mathematics achievement through numeration's effect on mathematical applications. These findings are consistent with explanation of skill transfer in terms of improvements in component subskills. In the case of the abacus, at least, the improvement of subskills seems to be a consequence of learning an effective mental analog, the mental abacus.

While the facilitative effect of abacus skill (via mental calculation skill) on the calculation subtest is not surprising, abacus skill's effect on numeration is noteworthy. The numeration test, it will be recalled, measures the child's conceptual understanding of the number system. We speculate that having two ways (conventional and abacus) to represent arithmetic calculation may give a child a more abstract and flexible understanding of the number system, in much the same way that knowledge of two languages can lead to higher levels of metalinguistic knowledge.

Both our studies lead us to conclude that it is inappropriate to explain skill transfer in terms of affective outcomes. In the first study, abacus skill affected perceived cognitive competence only indirectly, through skill's effect on achievement. In the second study, liking for mathematics was found to be an outcome of component subskill enhancement but was not directly related to mathematics achievement per se. Our rejection of affective explanations is strengthened by the lack of transfer from abacus skill to social studies achievement, by the near-zero correlation between reading achievement and liking for reading, and by previous research (e.g., Newman 1984; Scheirer and Kraut 1979).

We did find the expected indirect effect of abacus skill on spatial task performance. However, spatial ability did not relate to differences in reading achievement in this sample. We therefore did not find any evidence in this study that transfer to a superficially unrelated task might be due to enhancement of an underlying prerequisite ability.

Our findings suggest that the direction of impact is from skill to representation, from representation to subskill components, and from subskill components to performance on tasks that require those subskills. If this model describes the transfer to reading achievement as well as it does to mathematics achievement, then explanation of the transfer to reading requires that we locate component subskills that are improved by abacus training and that are also utilized in reading performance. Such an analysis requires more complete understanding than we have to date both of the cognitive prerequisites for reading performance and of the cognitive consequences of the mental abacus.

This is not to suggest that the transfer to reading achievement might not be demonstrated to result from improvement in a latent ability. We have merely demonstrated that spatial ability, at least as it is measured in this study, fails to provide a satisfactory explanation. An alternative might be found. For example, Suk-fong Tang (1985, personal communication) suggests that abacus practice may train students to generate mental imagery to represent and manipulate conventionalized symbols—an essential skill for students learning to read Chinese ideographs.

Further dangers exist in assuming that transfer is solely a consequence of improvement in component subskills. Even if component subskills can be identified and skill transfer can be analyzed accordingly, that does not imply that we know how a skill is built from its components. In fact, part of our demonstration has been that training in a skill can change the way that a task is mentally represented. We do not yet fully understand how this change in representation may affect strategies for tackling new tasks, representations of other tasks, or relations between tasks. Nevertheless, such understanding is fundamental to elaboration of an adequate theory of skill transfer (see Ferguson 1954, 1956, 1959).

This problem is illustrated by one of the surprising findings from our second study. Performance on number series tasks was found to be related to spatial task performance, but it was not directly affected by mental calculation. We had expected mental calculation to assist the hypothesis testing seemingly required for solution of number series problems. However, sequential testing and falsification of alternative solutions may be a relatively ineffective, slow, or confusing strategy for solving number series problems (Mynatt, Doherty, and Tweney

1978). Rapid generation of the correct solution instead may be facilitated by efficient use of intuitions rooted in manipulation of spatially based representations or analogs (see Kaufmann 1980; Sternberg 1985, chaps. 5, 6). Accordingly, spatial representations may direct attention to number system features that are particularly pertinent for problem solution (see Evans 1983; Olson and Bialystok 1982). What those representations are, how they are used, and how they are acquired are not yet clear. Nor do we understand how experiences, skills, and subsequent representations are linked to individual differences in the choice of problem-solving strategies for tasks like number series.

Our need to understand the impact of skills on mental representations and the subsequent impact of those representations on performance in other domains bears methodological implications for the study of intellectual performance. One received view in cognitive psychology holds that performance is to be understood as the expression of latent abilities (see Simon 1976). Accordingly, one should study skills from the inside out—from representations to performances and their contexts. However, if it is the case that mental representations themselves are a consequence of experience and that mental representations affect other skills, then it also becomes necessary to study skills from the outside in—from developmental contexts to consequent representations. Latent abilities cannot be separated from mental representations that are themselves consequences of skill.

Here we find a link between culture, society, and psychology. Cultures prescribe parameters for the appropriate organization of attention; societies provide contexts, rewards, and sanctions to direct and focus attention within those parameters; attention regulates the construction of experience (Csikszentmihalyi 1978). We were able to delineate consequences of abacus skill because abacus skill is valued in Chinese culture, rewarded by Chinese society, and fostered through Chinese education.

This significance of context also emerged in our demonstration that variations among Chinese children, as indexed by SES and Raven's Matrices, affected students' accomplishments. Socioeconomic status is a proxy for systematic variations in children's within-culture socialization histories (see Musgrave 1972). Raven's Matrices summarize variations in prior skill attainments (Chalip and Stigler, in press). Both measures show direct and indirect relations to achievement. We find, then, that a fully elaborated theory of skill transfer requires specification of the impact of between- and within-culture contexts on subsequent skilled performances.

The model that we suggested above therefore is incomplete. Transfer results from enhancement of component subskills; subskill enhancement results from mental representations that reflect experiences; experiences are prescribed by cultures and implemented as socialization. What cries out for further elaboration is the way that experiences and consequent mental representations both mold representations of related tasks and dictate strategies for tackling new tasks.

Notes

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1. A lower-bound estimate of grade reliability was calculated by correlating first- and second-semester grades separately within first and fifth grades. For first-grade math, $r = .876$; for first-grade reading, $r = .878$; for fifth-grade math, $r = .886$; for fifth-grade reading, $r = .887$. Thus, the school grades used in our study are quite reliable.

2. In order to meet the distributional assumptions of the LISREL procedure, distributions of the variables were checked. Grades showed a significant negative skew, and hours of training showed a significant positive skew. These were normalized using standard procedures of exponential and cube-root normalization, respectively.

3. The four paths constrained as equal were: the paths from reading and math grades in fifth grade to perceived cognitive competence; the paths from socioeconomic status to first-grade grades in math and reading; the correlations between math and reading grades at first and fifth grades; and the correlations between Raven's scores and reading and math grades at first grade. The equality of paths within each of these path pairs was tested using procedures provided by the LISREL program. Using these procedures, paths are constrained as equal, and the alteration in residual χ^2 is then used to test the path equalities. If the change in χ^2 is insignificant, it can be concluded that the paths are equal. When the four equality constraints were imposed on our data, we concluded that the path pairs were equal: $\chi^2(4) = 2.27$, $P > .5$.

4. The reduced sample size combined with the inclusion of highly correlated items made the LISREL procedures that were used in Study 1 impractical for analyzing the data from Study 2. Three-stage least-squares were used to estimate a path model (Schmidt 1976). These procedures provide an R^2 test of overall model fit, t-ratio tests of individual paths, F-ratio tests of path clusters, and F-ratio tests of path equalities. Intramodel correlations may be unspecified.

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